**CIMPA SCHOOL IN**

# Riemannian Geometry, pseudo-Riemannian Geometry and Mathematical Physics

**Marrakech 19 - 30 May 2008**

**Objective of the School:**

(1) Provide participants with an introduction to recent developments in Riemannian and pseudo-Riemannian geometries and their interactions with mathematical physics;
(2) Permit to participants who have taken part in CIMPA schools "Marrakech 2004" and "Eloued 2005" to strengthen the acquired formation;
(3) Reinforce the activities of GGTM[1] by establishing favorable conditions of rich collaborations between mathematicians from America and Europe and African mathematicians.

**During the first week of the school, the GGTM in collaboration with The city of Geometries of Meubeuge organize "Journées de Géométrie et Topologie".**[2]

**School location:**

University Cadi-Ayyad Marrakech,
Faculté des Sciences et Techniques
BP 549 Marrakech, Morocco.
Tel: 00 212 24 43 34 04/31 63
Fax: 00 212 24 43 31 70

**Periods of the School:**

19-30 May 2008.

[1]Groupement pour le Dévelopement de la Géométrie et de la Topologie au Maghreb.

[2]The detailed program of this activity is presented in an other document.
the address of GGTM is : *http://www.ggtm.uh2c.ma*

**Scientific directors for the school:**

M. Boucetta, Professor, University Cadi-Ayyad.
*<mboucetta2@yahoo.fr>*
P. Chrusciel, Professor, University François Rabelais Tours.
*<chrusciel@maths.ox.ac.uk>*
A. El Soufi, Professor, University François Rabelais Tours.
*<ahmad.elsoufi@univ-tours.fr>*
G. Zeghib, Research Director, ENS Lyon.
*<Abdelghani.ZEGHIB@umpa.ens-lyon.fr>*

**Coordinators of the School :**

**CIMPA representant :** Richard Grin *<grin@unice.fr>*

**Coordinators :** M. Boucetta & A. El Soufi

## 1. Scientific program

We present here the list of speakers and an abstract of their mini-courses.

**Abdelhak Abouqateb (University Cadi-Ayyad, Marrakech, ) and Daniel Lehmann (University Montpellier II)**:

*<abouqateb@fstg-marrakech.ac.ma>* & *<lehmann@math.univ-montp2.fr>*

Title: *Principal bundles and connections*

Abstract: In this course, we will introduce the basic material on principal bundles and connections in view of its applications in mathematical physics.

**Aziz EL Kacimi (University of Valenciennes)** :
*<Aziz.ElKacimi@univ-valenciennes.fr>*

Title: *De Rham cohomology as example of a topological invariant*

Abstract: The following question is natural in Differential Topology: how to recognize that two manifolds are homeomorphic or not? One of the main tools used to "answer" this question is de Rham *cohomology*. This is a vector space $H^*(X)$ associated to any differentiable manifold $X$ such that if $X$ and $Y$ are homeomorphic then $H^*(X)$ and $H^*(Y)$ are isomorphic! We say that $H^*(X)$ is a *topological invariant*. The goal of these lectures is to give an elementary introduction to this theory and simple examples to illustrate its power and its usefulness.

**Helga Baum (Humboldt-Universität Berlin)** :
*<Baum@mathematik.hu-berlin.de>*

Title: *Introduction to holonomy of Riemannian and Lorentzian manifolds*

Abstract: In this course, we will introduce basic material on holonomy groups of (pseudo)-Riemannian manifolds. After presenting the long known classification results in the Riemannian case, we will focus on recently obtained classification results for Lorentzian holonomy groups and discuss local and global Lorentzian metrics with special holonomy. We will also explain the relation between holonomy groups and special spinor fields.

**F. Béguin (University Paris VI, France) and A. Zeghib (ENS Lyon, France)**:

*<Francois.Beguin@math.u-psud.fr>* & *<Abdelghani.ZEGHIB@umpa.ens-lyon.fr>*

Title: *Constant curvature spacetimes*

Abstract: The study of constant curvature spacetimes allows one to introduce certain notions and problems of Global Lorentzian Geometry in a quite simplified context. Indeed, constant curvature spacetimes may exhibit a very rich topology and global geometry, but have — by definition — a trivial local geometry.
During this course, we shall present some classical concepts of General Relativity (e.g. *past* and *future* of a set, *causal spacetime*, *globally hyperbolic spacetime*, *time function*, *conformal boundary of a spacetime*, *initial singularity of a spacetime*, *horizon of a black hole*) and illustrate these concepts in the context of constant curvature spacetimes. Simultaneously, we will explain how the notions of *developing map* and *holonomy group* allow one to give a complete description of large classes of constant curvature spacetimes. A certain number of open questions arise from this description.
This course — which we hope to be interesting by itself — can also be considered as an introduction to the course on *Bianchi cosmological models*.

**F. Béguin (University Paris VI, France) and A. Zeghib (ENS Lyon, France)**:

*<Francois.Beguin@math.u-psud.fr> & <Abdelghani.ZEGHIB@umpa.ens-lyon.fr>*

Title: *Bianchi Cosmological models*

Abstract: Bianchi cosmological models are *spatially homogenous spacetimes*. More precisely, they are manifolds of the form $M = I \times G$ where $I \subset \mathbb{R}$ is an interval and $G$ is a Lie group, endowed with a lorentzian metric of the form $-dt^2 + g_t$ where $(g_t)_{t\in I}$ is a family of left-invariant Riemannian metrics on $G$.
The physical content of a spacetime $M$ is encoded by a non-linear PDE on its lorentz metric: the so-called Einstein equation. The first goal of our course will be to explain why, in the context of Bianchi cosmological models, this PDE reduces to a second order *ordinary differential equation* on the family of metrics $(g_t)_{t\in I}$. Then we will show that, for certain Lie groups $G$, this equation has a rather simple dynamics, whereas in some other cases, it exhibits a very rich and chaotic dynamics (which is still not fully understood).
We think that the main interest of the course on Bianchi Cosmological Models is that it stands at the crossroad of a wide variety of mathematical and physical fields : Cosmology, Global Riemannian and Lorentzian Geometry, PDE's, Dynamical Systems Theory, Lie Groups Theory,...

**Piotr Chrusciel (Université François Rabelais, Tours)**:

*<chrusciel@maths.ox.ac.uk>*

Title: *Introduction to black holes*

Abstract: In this mini-course we will introduce and briefly describe the Schwarzschild, Kerr, and Emparan-Reall black holes. We will present the key theorems of black hole theory: topological censorship, area, and topology theorems. We will prove uniqueness of static vacuum black holes.

**Evans Harrell (Georgia Institute of Technology, Atlanta)**

*<harrell@math.gatech.edu>*

Title: *Mathematics of quantum mechanics on thin structures.*

Abstract: Microelectronic devices known as quantum wires and waveguides are in production on scales where the electrons are subject to quantum effects and described with the Schrödinger equation. These devices are macroscopic in one or two dimensions but on the scale of nanometers in at least one dimension.
The course will begin a discussion of some current research in nanophysics laboratories and the mathematical models used in the field. The idealization of a thin domain in quantum mechanics as a restriction to a submanifold is one of the more important models, so careful asymptotics will be carried out for this model. The geometry of the submanifold will contribute explicitly to an effective potential in the limiting Schrödinger equation.
Spectral theory of elliptic PDEs will be reviewed and applied to Schrödinger operators on manifolds. The effect of the geometry on the eigenvalues (energy levels) will be investigated with a variety of techniques, some variational and some purely algebraic, including "sum rules" for the spectra and "universal bounds" on eigenvalues. Special features of the Schrödinger operators of quantum wires and waveguides will be considered from physical, geometric, and analytical points of view.
Connections will be made to isoperimetric theorems, Weyl asymptotics (asymptotics for high energies), and semiclassical approximation. The latest known results of these types will be explained.

**Aziz Ikemakhen (Université Cadi-Ayyad, Marrakech)**

*<ikemakhen@fstg-marrakech.ac.ma>*

Title: *Introduction to spinorial geometry*

Abstract: In this courses, we introduce Cliffords algebras and their representations, the spin groups and their representations. We introduce the notion of spin structure and spinc structures on a manifolds and constructs the spinorial bundles and their canonical connections.

**Jim Isenberg (University of Oregon)**

*<jim@newton.uoregon.edu>*

Title: *Finding Solutions of the Einstein Constraint Equations*

Abstract: The initial value approach is the most widely used method for finding and studying solutions of the Einstein gravitational field equations, and the first step in carrying out an initial value construction is to solve the Einstein constraint equations. In this set of lectures,

we discuss some of the methods which have been developed for studying the constraint equations and their solutions. We particularly focus on the conformal and conformal thin sandwich methods, and on gluing techniques.

**Frank Pacard (University Paris XII)**

*<pacard@univ-paris12.fr>*

Title: TBA

## 2. Schedule

**The registration will take place Monday 19 May from 8:30 to 9:30.**

### First week

| **Monday** May 19 | **Tuesday** May 20 | **Wednesday** May 21 | **Thursday** May 22 | **Friday** May 23 |
|---|---|---|---|---|
| 9:30-11:00 EL Kacimi | 8:45-10:15 Baum | 9:00-10:00 François Recher GGTM | 8:45-10:15 Pacard | 8:45-10:15 Ikemakhen |
| 11:00-11:30 Coffee Break | 10:15-10-45 Coffee Break | 10:00-11:00 Reception GGTM | 10:15-10-45 Coffee Break | 10:15-10-45 Coffee Break |
| 11:30-12:30 Sergiescu GGTM | 10:45-11:45 Ikemakhen | 11:00-12:00 Valerio Vassallo GGTM | 10:45-11:45 EL Kacimi | 10:45-11:45 Hijazi GGTM |
| | 11:45-12:45 Gamara GGTM | | 11:45-12-45 Melnick GGTM | |
| Lunch Break | Lunch Break | Lunch Break | Lunch Break | Lunch Break |
| 15:00-16:30 Baum | 15:00-16:30 Pacard | 15:00-16:30 Pacard | 15:00-16:30 Lehmann/Abouqateb | 15:00-16:30 Baum |
| 16:30-18:00 Lehmann/ Abouqateb | 16:30-18:00 EL Kacimi | 16:30-18:00 Lehmann/ Abouqateb | 16:30-18:00 Ikemakhen | 16:30-18:00 Lehmann/ Abouqateb |

## Second week

| **Monday** May 26 | **Tuesday** May 27 | **Wednesday** May 28 | **Thursday** May 29 | **Friday** May 30 |
|---|---|---|---|---|
| 9:00-10:30 Béguin/Zeghib | 9:00-10:30 Harrel | 9:00-10:30 Chrusciel | 9:00-10:30 Harrel | 8:45-10:00 Béguin/Zeghib |
| 10:30-11:00 Coffee Break | 10:30-11:00 Coffee Break | 10:30-11:00 Coffee Break | 10:30-11:00 Coffee Break | 10:30-11:00 Coffee Break |
| 11:00-12:30 Isenberg | 11:00-12:30 Chrusciel | 11:00-12:30 Béguin/Zeghib | 11:00-12:30 Béguin/Zeghib | 10:15-11:45 Harrell |
| Lunch Break | Lunch Break | Lunch Break | Lunch Break | Lunch Break |
| 15:00-16:30 Chrusciel | 15:00-16:30 Isenberg | 15:00-16:30 Free afternoon | 15:00-16:30 Isenberg | |
| 16:30-18:00 Béguin/Zeghib | 16:30-18:00 Béguin/Zeghib | 16:30-18:00 Free afternoon | 16:30-18:00 Béguin/Zeghib | |

### 3. Riad Omar

The following participants will stay at

**Riad Omar**
22, rue Bab Agnaou - Médina - Marrakech
Phone: 00 212 24 44 56 60
Url www.riadomar.com

| Conférencier | Arrivée | Départ |
|---|---|---|
| Helga Baum | 18 | 25 |
| Chrusiciel | 20 | 29 |
| EL Soufi | 18 | 31 |
| Jim Isenberg | 24 | 31 |
| Frank Pacard | 19 | 23 |
| Béguin | 19 | 29 |
| Zéghib | 18 | 30 |
| Harrel | 23 | 31 |
| Grin | 20 | 29 |

## 4. Hotel Farouk

The following participants will stay at

**Hôtel Farouk**
- 66, Avenue Hassan II - Guéliz - Centre Ville - Marrakech
Phone: 00 212 24 43 19 89
Url www.hotelfarouk.com

Hamid Abchir
Malek Fereshteh Azhar
Abbas Majed Alshimari
Iftime Mihaela
Ouakkas Seddik
Tagne Wafo Roger
Arbab Brahim
Benmoussa Tayeb
Youssef Nabil Labib
Moukaddem Nazih
Hathout Fouzi
Dida Hamou
Ben Ahmad Ali
Belkhirat Abdelhadi
Mdjerab Ouardia
Smai Djamel
Benmouhoub Naima

Jbilou Asma
Bahayou Amine
Oussalah Malika
Laurent Bakri
Chergui Brahim
Ilham Rouchdi
Fouzia El Wassouli
Mamouni My Ismail
Ekdiha Mohamed
Jalal Lassiri

Ganbouri Bousselham
Mansouri Wadie
Benroumane Abderazzak
Bouayadi Mohamed

## 5. Trip to Atlas Mountains

**Saturday 24 May we will organize a trip to Atlas Mountains.** The registration can be done 19 May.

## 6. More informations

Marrakech or Marrakesh (????? Marrakesh), known as the "Red City" or "Al Hamra," is a city with a population of 1,036,500 (as of 2006) in southwestern Morocco, near the foothills of the Atlas Mountains.
Marrakesh is the second largest city in Morocco after Casablanca, and was known to early travellers as "Morocco City." Prior to the advent of the Almoravids in the 11th century, the area was ruled from the city of Aghmat. The Almoravid leader, Abu-Bakr Ibn-Umar decided Aghmat was becoming overcrowded and decided to build a new capital. Being a nomad from the Sahara Desert, he decided to build it in a plain, away from the mountains and rivers, and chose the site of Marrakech as being in neutral territory between two tribes who were vying for the honor of hosting the new capital. Work started in May 1070, but Abu-Bakr was recalled to the Sahara to put down a rebellion in January 1071, and the city was completed by his deputy and eventual successor Yusuf ibn Tashfin[2]. The city experienced its greatest period under the leadership of Yacoub el Mansour, the third Almohad sultan. A number of poets and scholars entered the city during his reign, and he began the construction of the Koutoubia Mosque and a new kasbah.
Prior to the reign of Moulay Ismail, Marrakech was the capital of Morocco. After his reign, his grandson moved the capital back to Marrakech from Meknès.
For centuries Marrakesh has been known for its 'seven saints'. When sufism was at the height of its popularity, during the reign Moulay Ismail, the festival of the 'seven saints' was founded by Abu Ali al-Hassan

al-Yusi at the request of the sultan. The tombs of several renowned figures were moved to Marrakesh to attract pilgrims in the same way Essaouira did at that time with its Regrega festivals. The 'seven saints' (sebaa rizjel) is now a firmly established institution, attracting visitors from everywhere. The seven saints include Sidi Muhammad al-Jazuli, Sidi Abu al-Qasim Al-Suhayli and Cadi Ayyad ben Moussa.
Marrakech was dominated in the first half of the 20th century by T'hami El Glaoui, Lord of the Atlas and Pasha of Marrakech.
Demographics Marrakech had a population of 1,036,500 in 2006.

**Trips from Marrakech**

Many tourists take a trip from Marrakech to visit the valley of the Ourika River in the Atlas Mountains [3]or the valley of the Draa River in the south near the Sahara desert, but also to Middle Atlas Mountains: Waterfalls of Beni Mellal, and to the Essaouira on the Atlantic ocean.

**Transport**
Menara International Airport serves as the main airport for the city and receives flights from Europe and neighboring Arab countries.
A toll-paying motorway connects Marrakech with Casablanca.
CTM coaches (intercity buses) and various private lines run services to most notable Moroccan towns as well as a number of European cities, from the Gare Routière on Rue Bab Doukkala in downtown Marrakech.
Marrakesh is the southern terminus of the ONCF, the Moroccan railway network, and Marrakesh is well served by trains heading to Tangier, Rabat, Casablanca, and Fez. The train station is located on Avenue Hassan II. [4]
The ONCF owned "Supratours" bus company serves towns not served by the train. The bus timetable coordinates with the train timetable and the bus terminal is right beside the station.
**A trip form Menara Airport to the hotel in a Taxi cost 50 to 100DH (1 euros=10 DH). The normal trip in the town depends on the distance. For example, a trip from Riad Omar to the faculty cost between 10 and 15 Dh.**

[3]**Saturday 24 we will organize a trip to Atlas Mountains.**
[4]**The Farouk Hotel is located in the same Avenue.**